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# STRAIN MAPPING BY HIGH ANGULAR RESOLUTION EBSD (AKA HR-EBSD)

**Acknowledgments:** Angus Wilkinson, Angus Kirkland, Fionn Dunne, Jun Jiang, Vivian Tong, David Wallis, Lars Hansen, Aimo Winkelmann

# Outline

- ⦿ EBSD vs HR-EBSD
- ⦿ Examples
- ⦿ Strain and deformation
- ⦿ HR-EBSD fundamentals
- ⦿ Accuracy, precision and sensitivity
- ⦿ Pattern Remapping
- ⦿ Summary

# EBSD vs HR-EBSD

	EBSD	HR-EBSD
<b>Absolute Orientation</b>	~2°	No
<b>Misorientation</b>	~0.1 to 0.5°	~0.006°
<b>Deformation –</b>		
<b>GNDs @ 1μm step</b>	$> 3 \times 10^{13}$	$> 3 \times 10^{11}$
<b>GNDs @ 100nm step in lines / m<sup>2</sup> (b = 0.3nm)</b>	$> 3 \times 10^{12}$	$> 3 \times 10^{10}$
<b>Relative elastic strain</b>	No	Deviatoric strain $\pm 1 \times 10^{-4}$
<b>Relative residual stress (Type III – within grain)</b>	No	Anisotropic Hooke's law $\pm 20$ MPa  (E=200GPa)
<b>Example tasks:</b>	Microstructure, Texture, Grain size, etc.	<b>Deformation</b> i.e. elastic strain, misorientation & residual dislocation content



# Strain and deformation

# Strain and deformation

- ⦿ Two types of strain tensor
  - Elastic (i.e. stress)
  - Plastic
- ⦿ Elastic leads to a change in bond length / angle  
→ HR-EBSD!
- ⦿ Plastic more tricky...

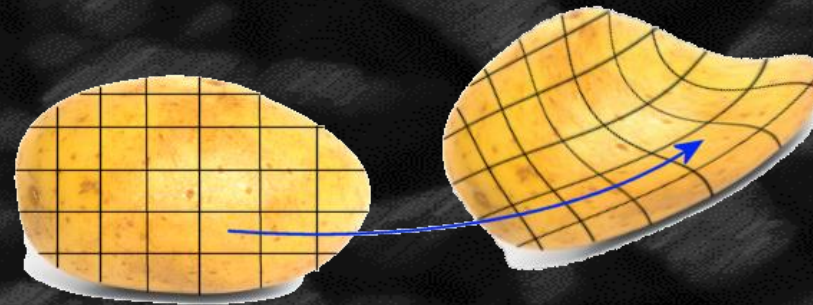
# Strain and deformation

Plastic shape change  
due to slip etc.  
Lattice remains un-deformed

$$\mathbf{F} = \mathbf{F}^e \mathbf{F}^p$$

Elastic shape change  
→ Bond stretch etc.  
→ Lattice orientation  
due to slip etc.

$$\mathbf{F} = \begin{pmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{pmatrix} = \begin{pmatrix} \frac{dx}{dX} & \frac{dx}{dY} & \frac{dx}{dZ} \\ \frac{dy}{dX} & \frac{dy}{dY} & \frac{dy}{dZ} \\ \frac{dz}{dX} & \frac{dz}{dY} & \frac{dz}{dZ} \end{pmatrix} + \mathbf{I}$$

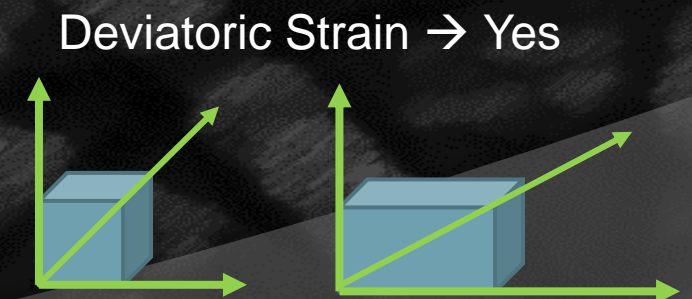
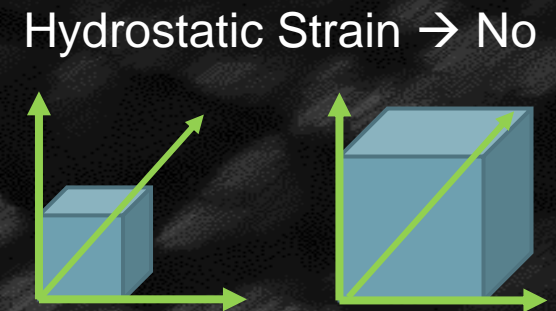
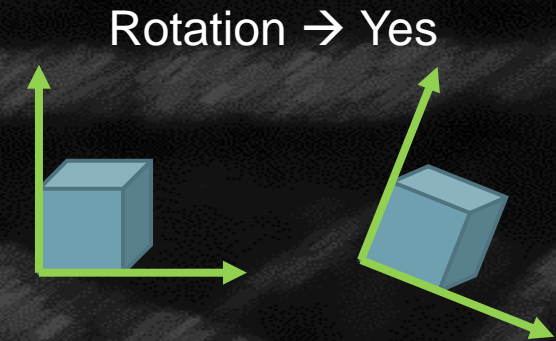


Strain is fundamentally a 2<sup>nd</sup> rank tensor

Many metrics are proxies  
→ use with care!

# HR-EBSD and $F^e$

- Follow change in interplanar angles within the diffraction (with high precision)
- Apply simple geometry and extract  $F^e$
- Split  $F^e$  into elastic strain (deviatoric) & lattice rotation
- Use lattice rotation gradients to evaluate GNDs (a symptom of plastic strain)

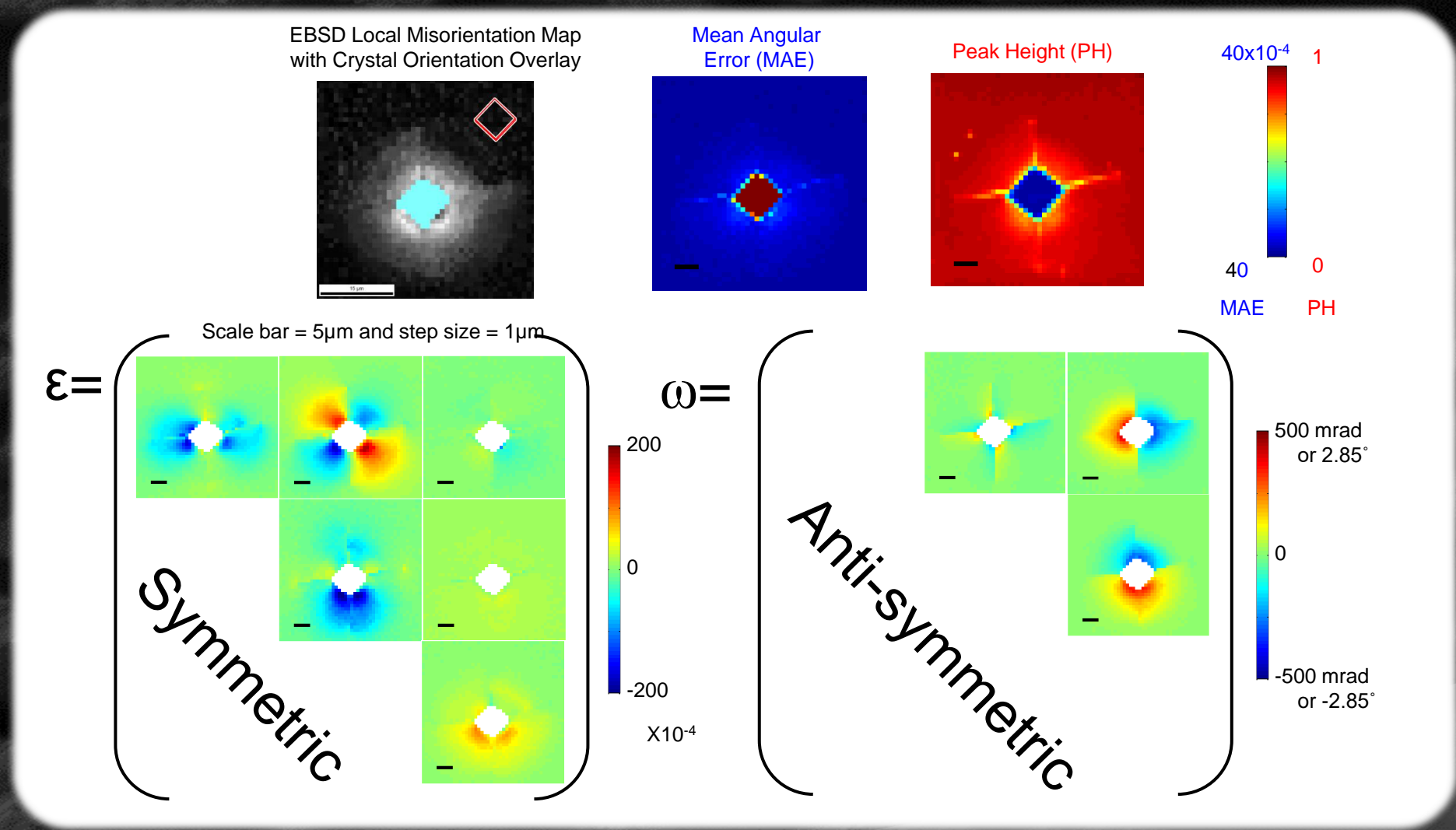




Examples

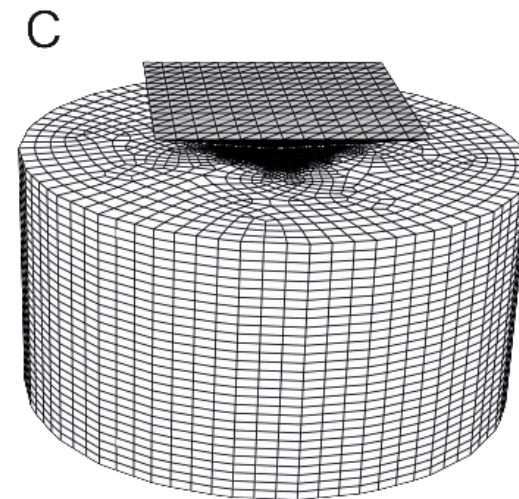
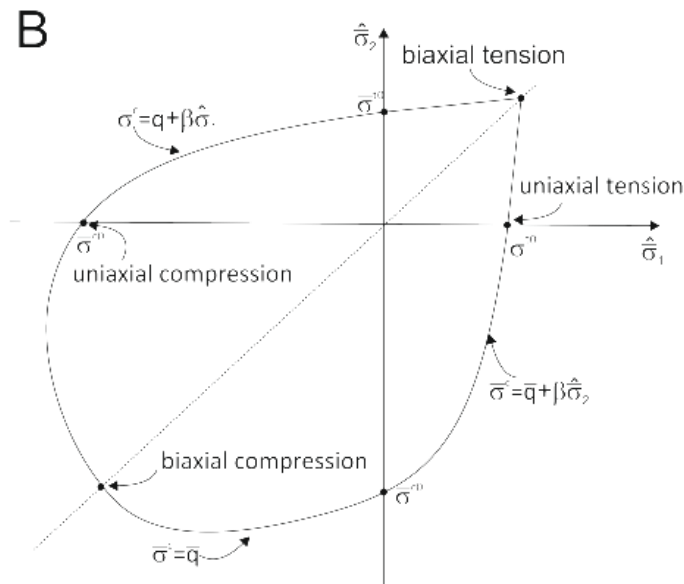
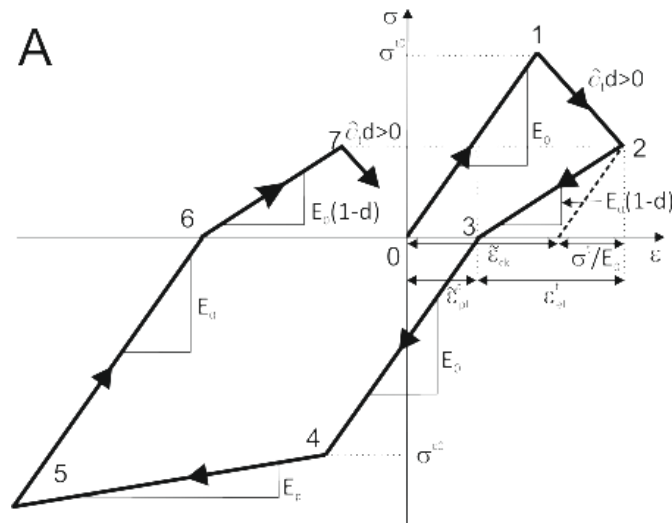


# Strain & rotation



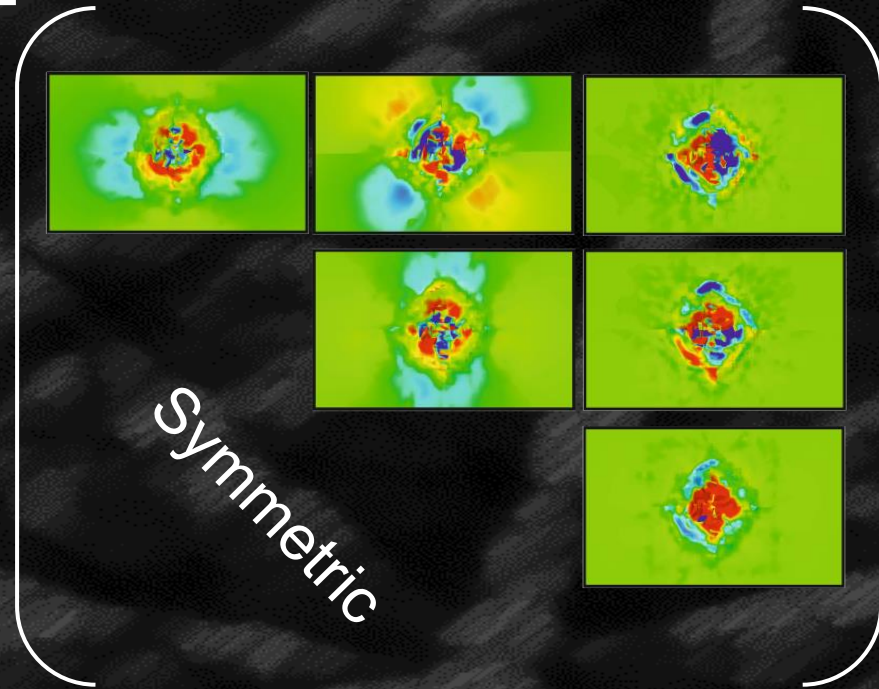
# Comparison with models

- 50g force
- Concrete damage model + cohesive zones along  $\langle 110 \rangle$



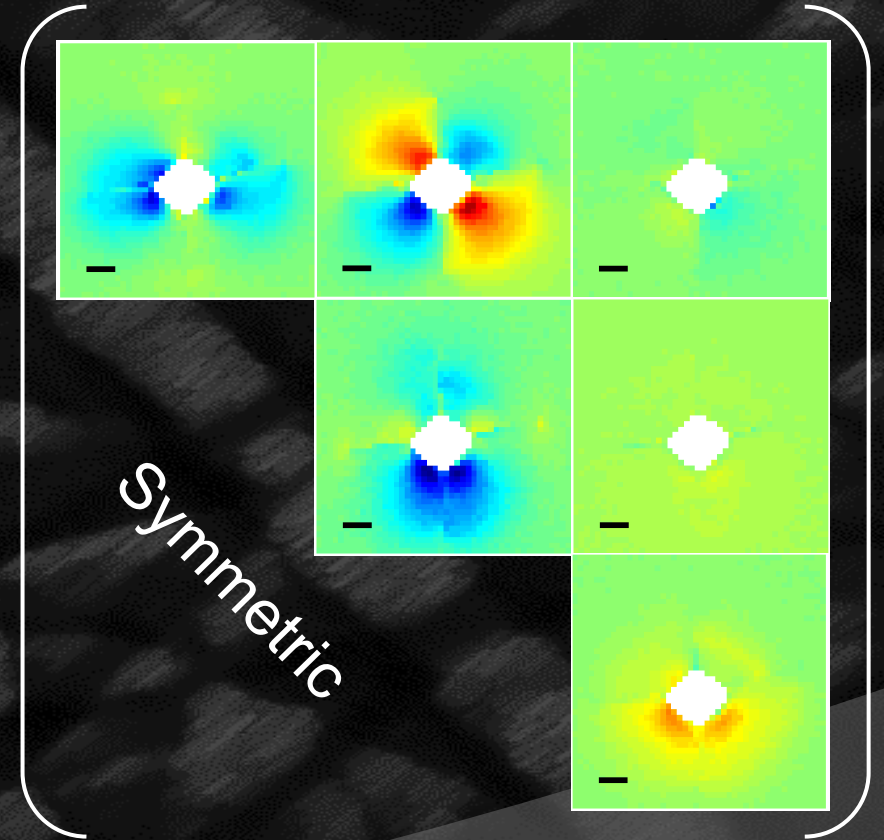
# Comparing Strain Fields

$\epsilon =$



$\times 10^{-4}$

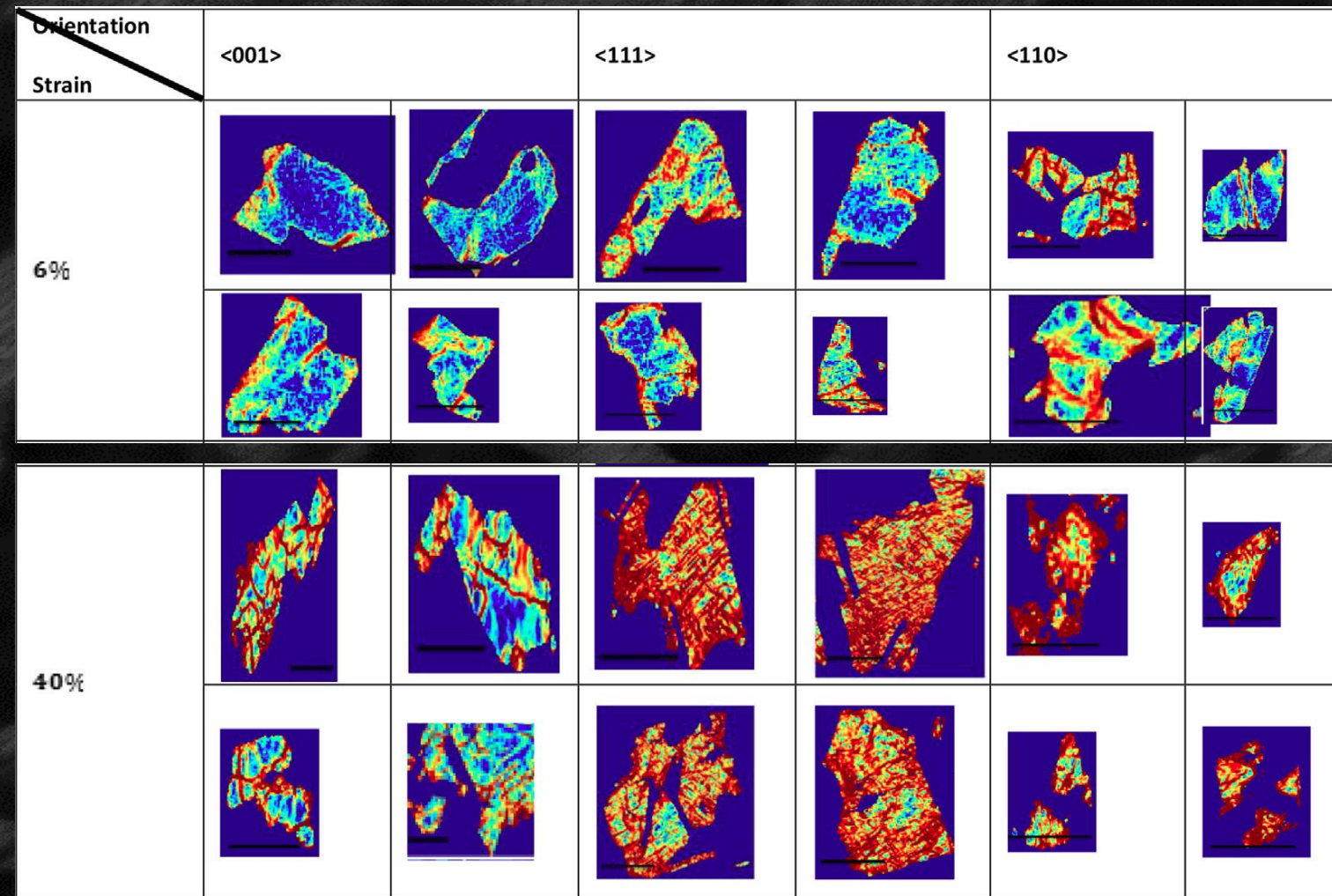
$\epsilon =$



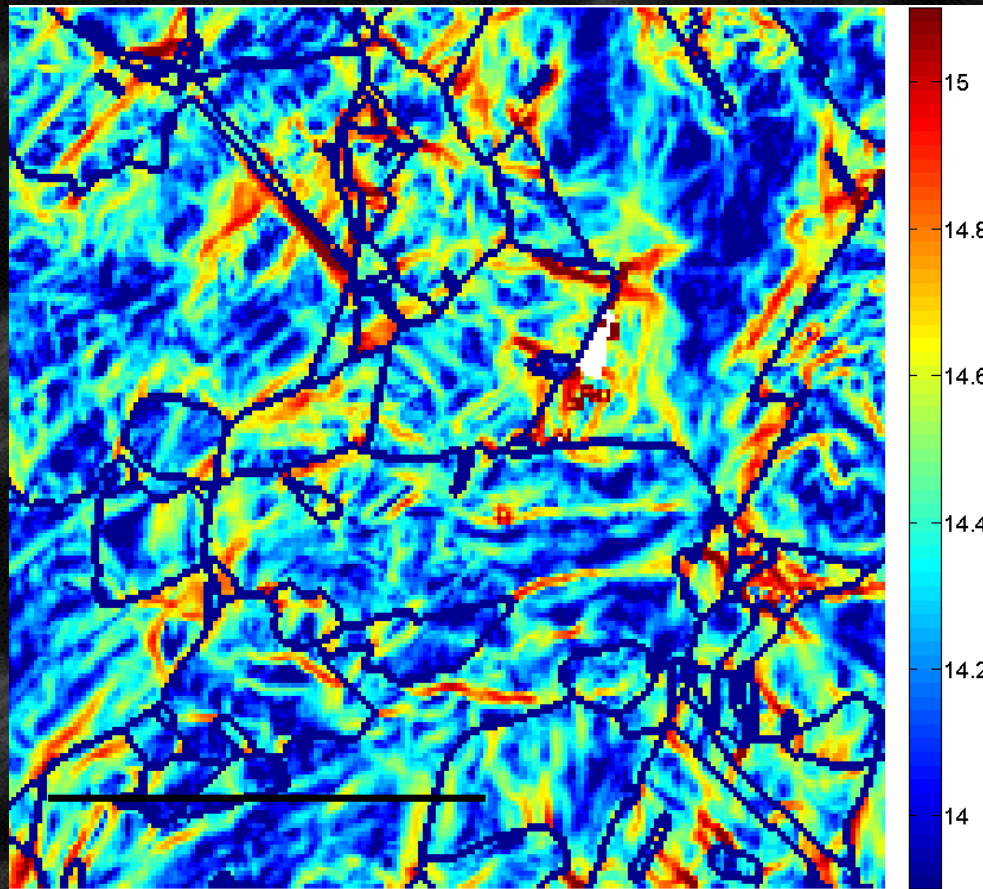
$\times 10^{-4}$

# Microstructure – dislocation structure correlations (Cu)

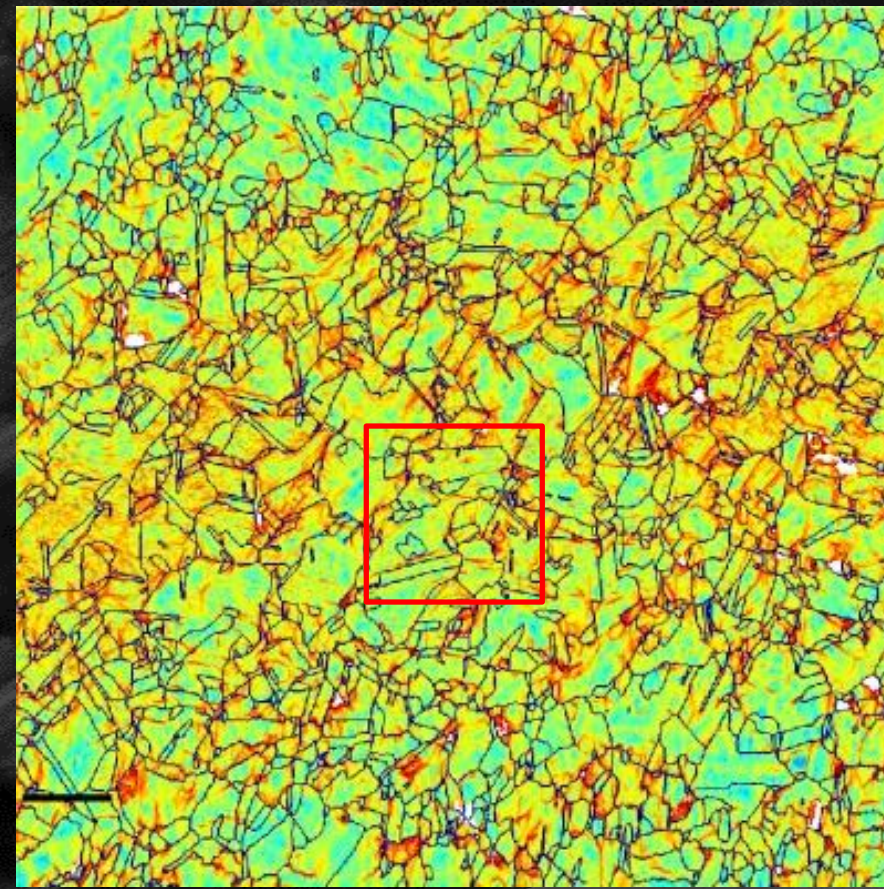
- Evolution of dislocation structure with uniaxial strain
  - Avoids thin films
  - Cover large areas
  - Access many grains
- Correlate with tensor quantities
  - Schmid factor, Taylor factor etc.
- Can also compare with crystal plasticity models



# Big data...

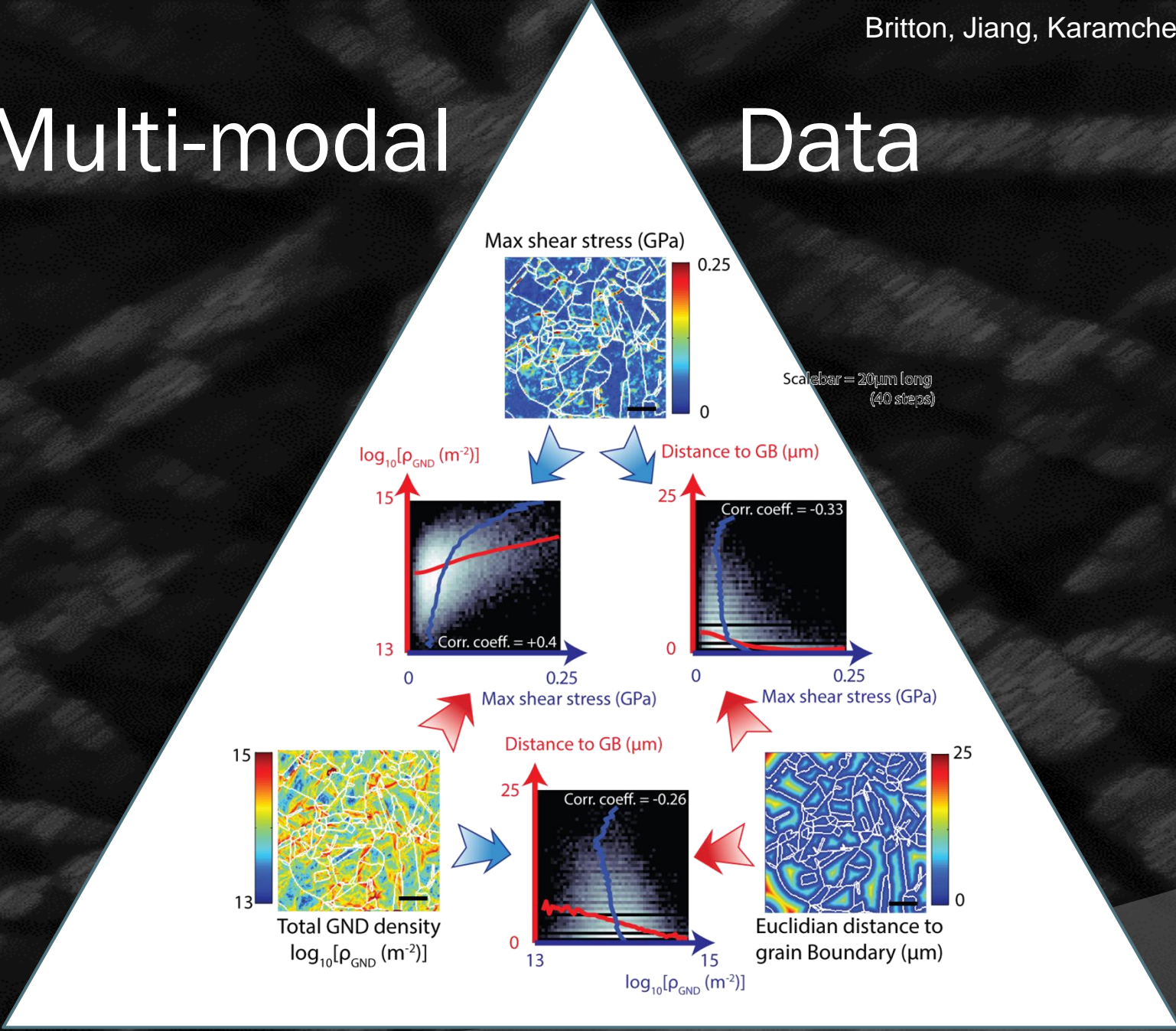


~110 $\mu$ m square = 30-60 grains

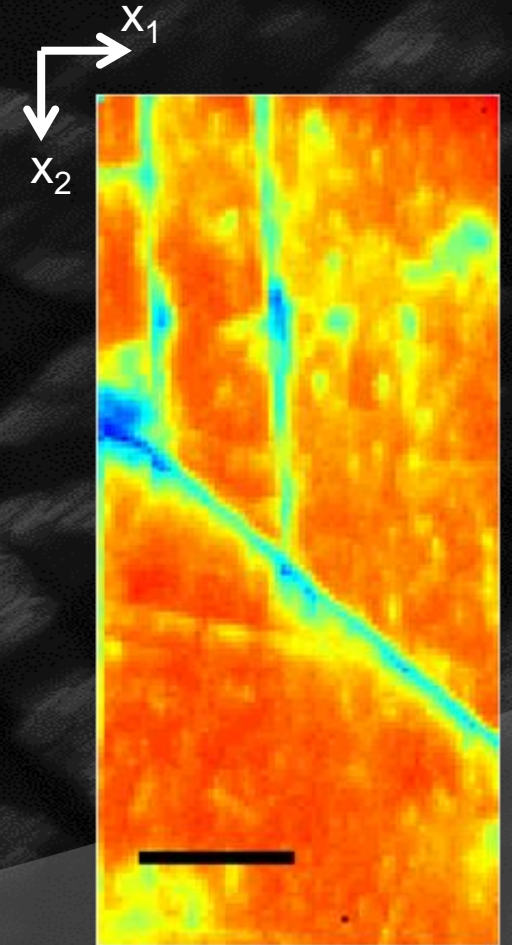
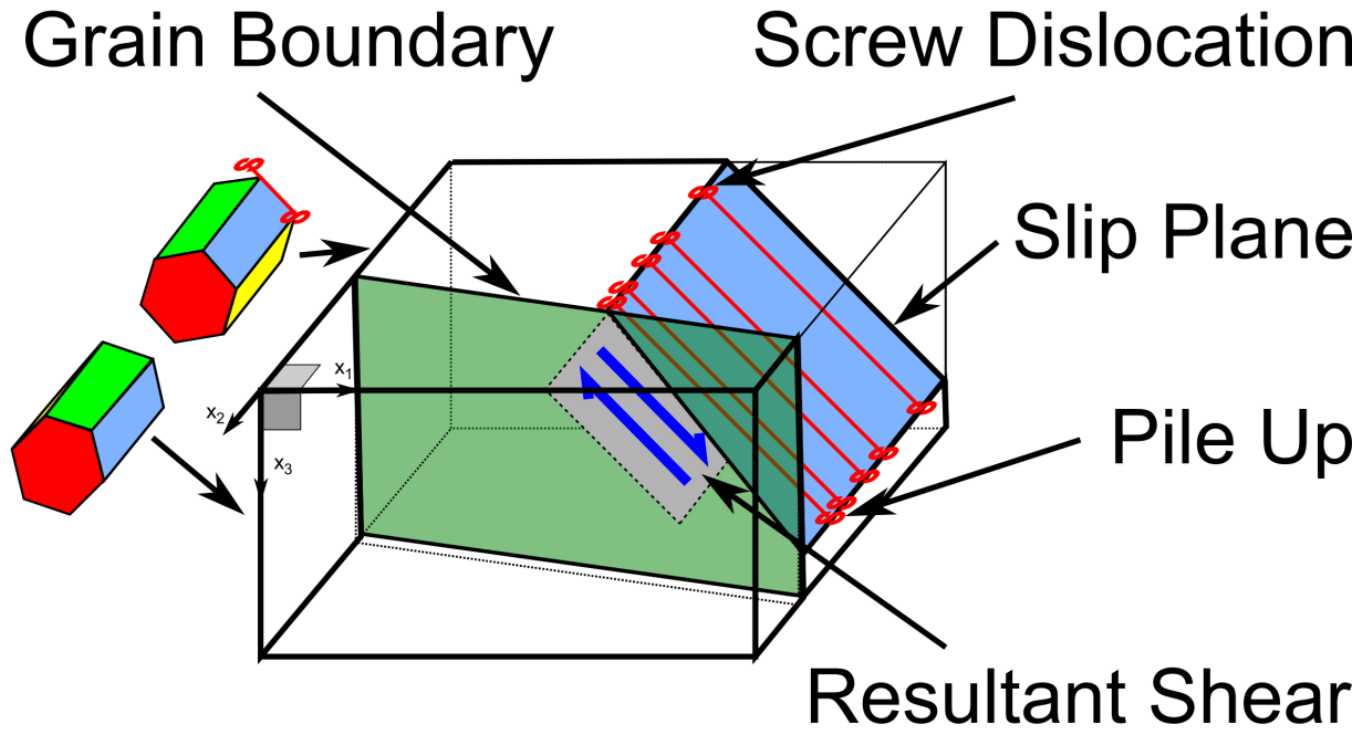


~500 $\mu$ m square = 1500 grains

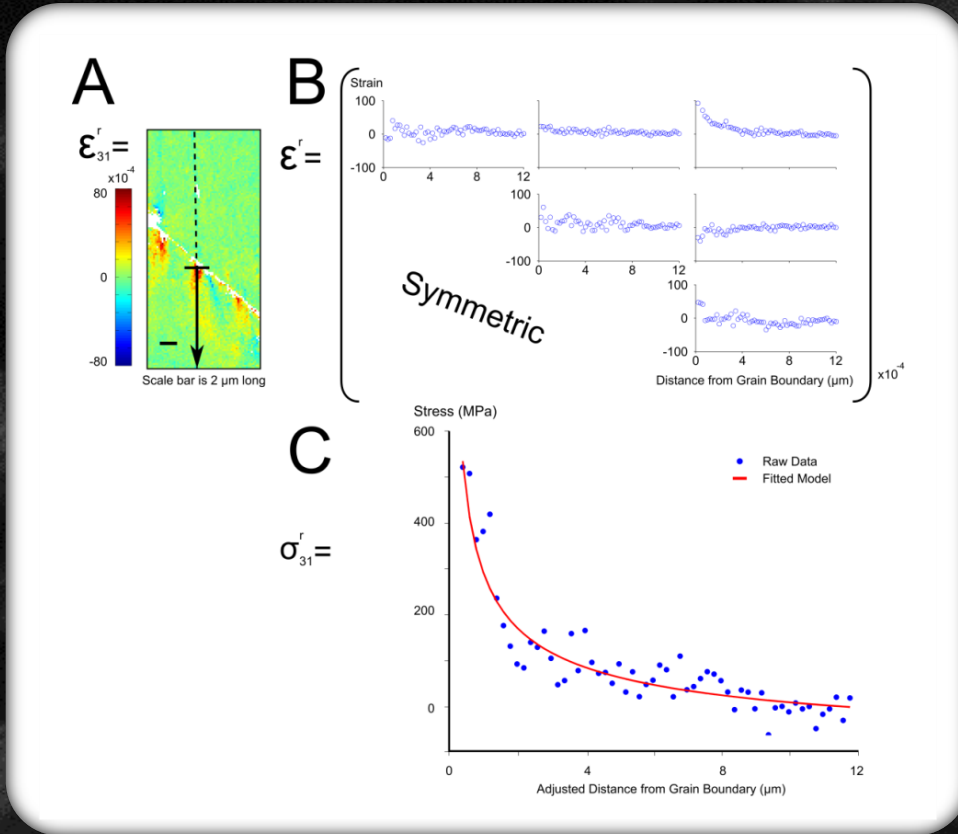
# Multi-modal Data



# New insight – g.b. strength

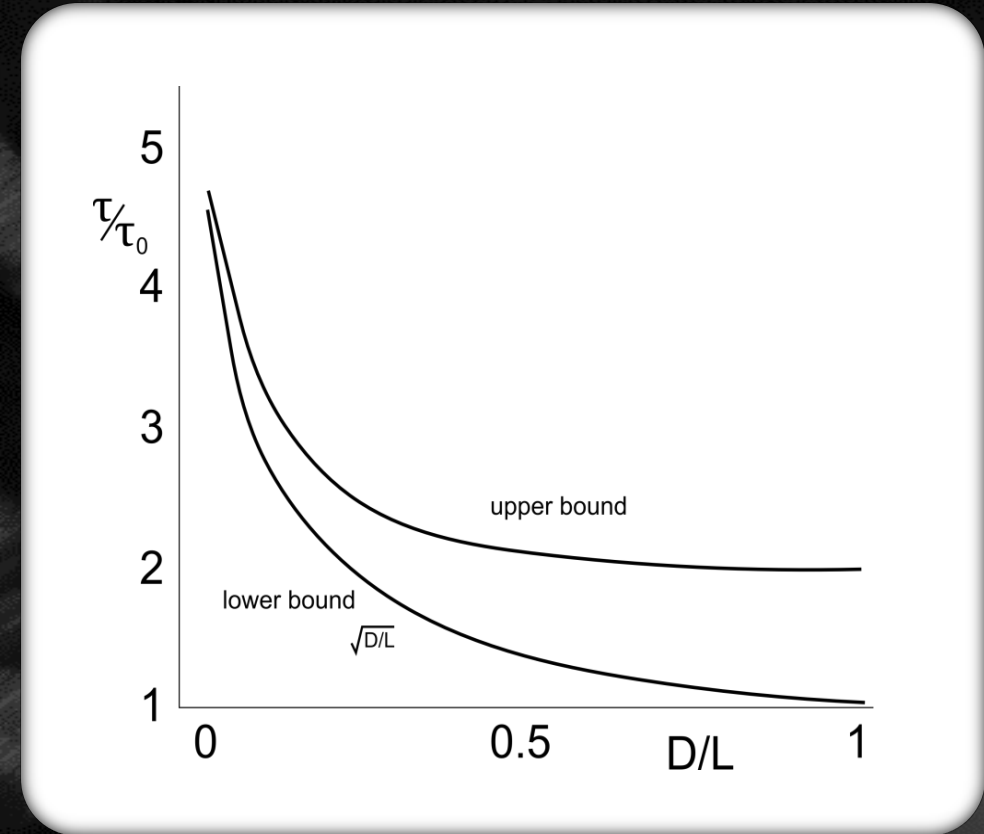


# Verifying Eshelby, Frank and Nabarro



Measurement of stress and strain near g.b.

fitting:  $\sigma_{31} = A + K / \sqrt{D}$   
 $K = 0.42 \text{ MPa}\sqrt{\text{m}}$



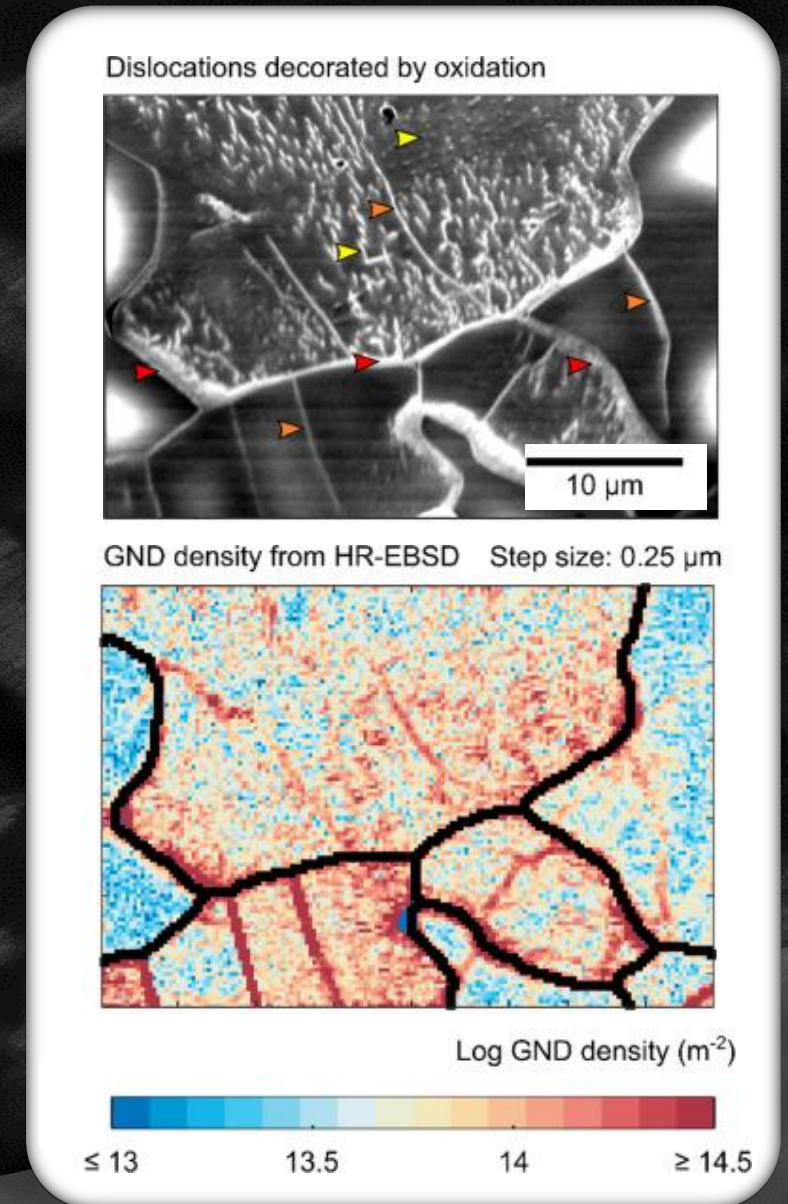
Normalised shear stress vs normalised distance from a screw dislocation pile up

Redrawn from Eshelby, Frank, Nabarro (1951)



# HR-EBSD Rocks

- ⦿ All we need are good EBSD patterns...
- ⦿ Other material systems can be explored
  - e.g. Metals, Semiconductors, Rocks, Intermetallics etc.
- ⦿ Example here – dislocations in olivine
  - Good correlation between dislocation decoration (oxidation) & GND measurements
  - Important for understanding earth formation

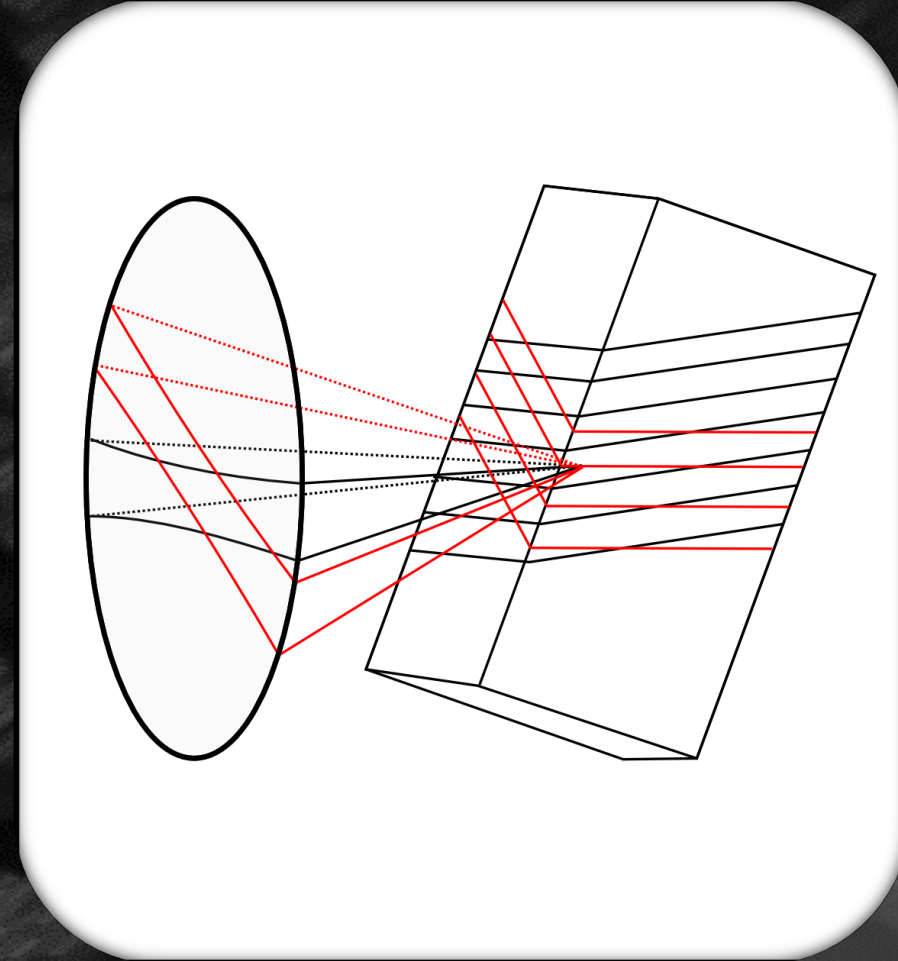


The background of the slide is a grayscale HR-EBSD (High-Resolution Electron Backscattered Diffraction) pattern. It features a complex, repeating pattern of diffraction spots and lines, characteristic of a polycrystalline material. The pattern is centered and occupies most of the frame. A large, semi-transparent dark gray circle is overlaid on the right side of the image, partially obscuring the HR-EBSD pattern. The text 'HR-EBSD Fundamentals' is centered horizontally and vertically within the visible area of the HR-EBSD pattern.

# HR-EBSD Fundamentals

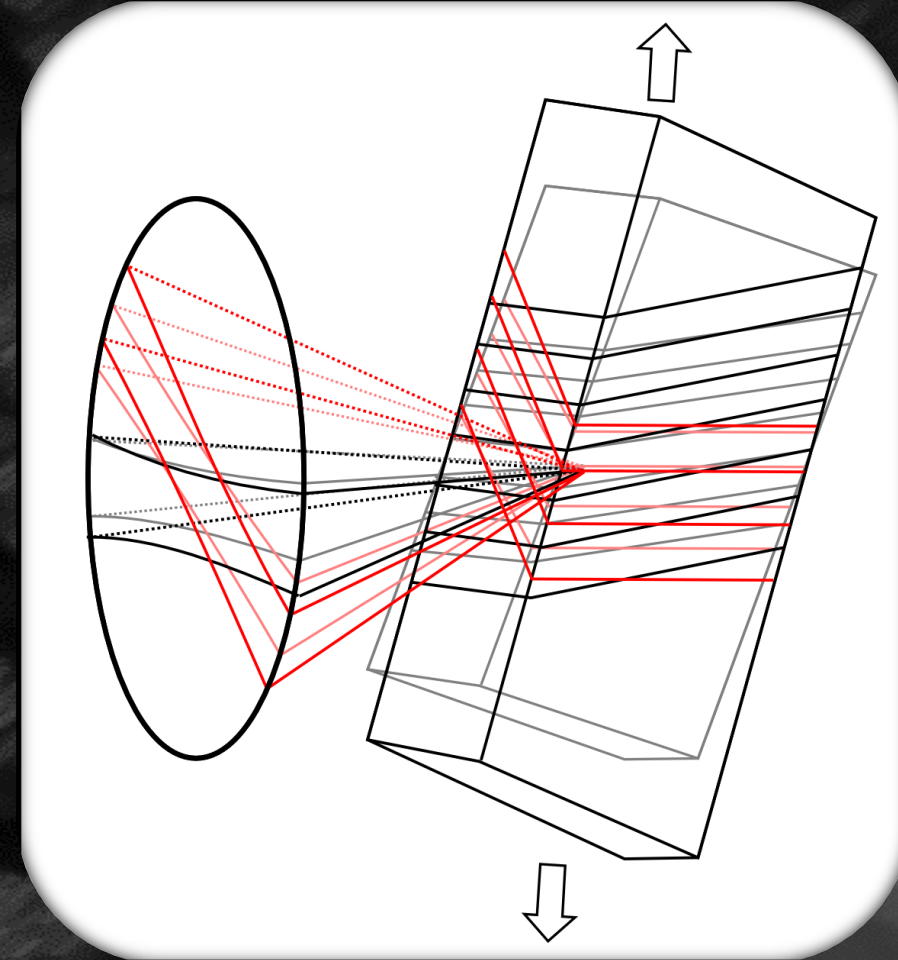
# Accessing Elastic Strain – HR-EBSD

- EBSD pattern = direct projection of lattice planes



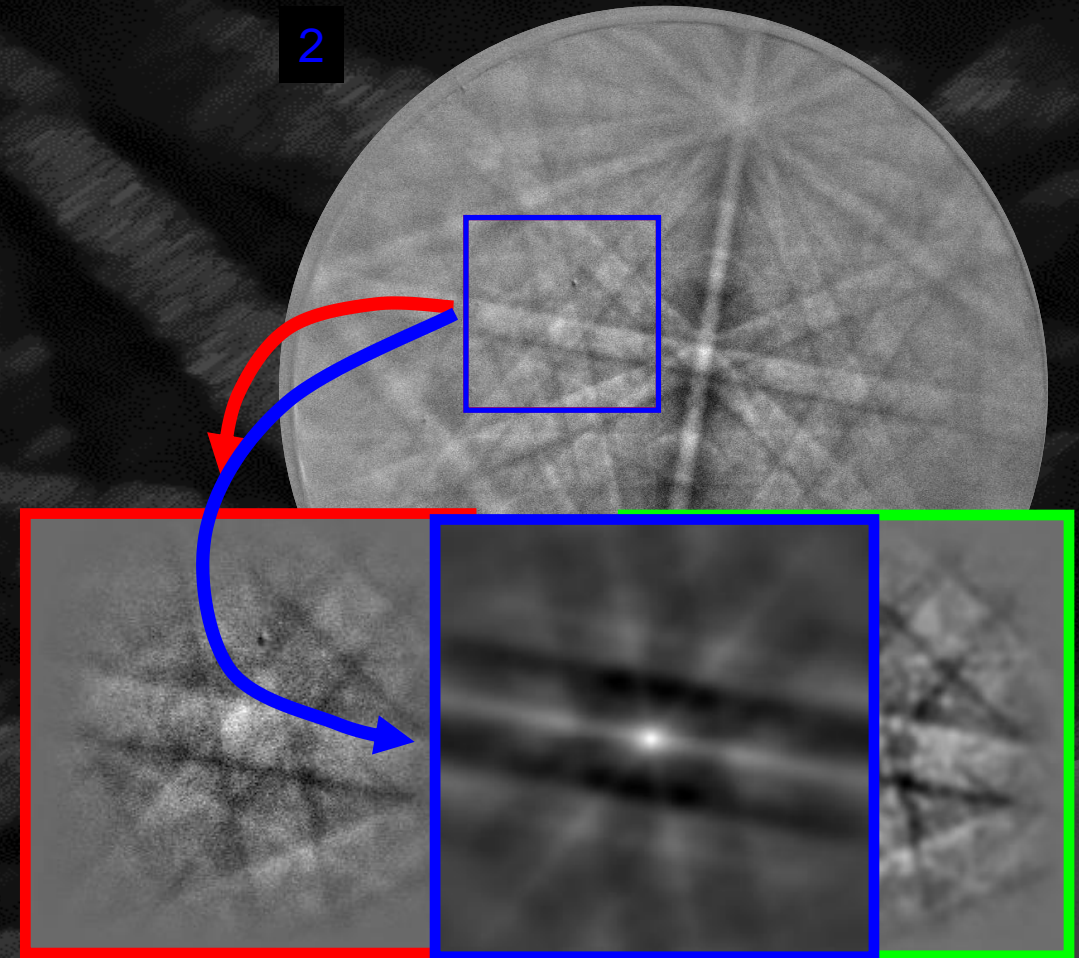
# Accessing Elastic Strain – HR-EBSD

- EBSD pattern = direct projection of lattice planes
- Strain of crystal = movement of bands

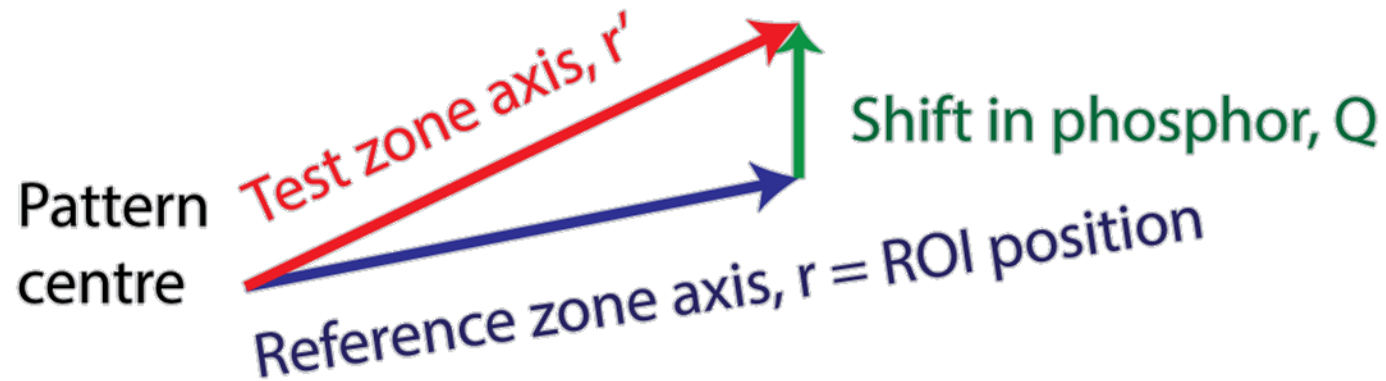


# Measuring shifts with image correlation

- 20+ ROI used (offline)
  - Select ROIs  $\rightarrow$  FFT
  - Apply filter
- Compare unstrained (1) vs strained (2) pattern
  - Upsample peak in XCF of ROI
    - xshift= -6.06 (pixels)
    - yshift= -4.59
- ‘Just’ an educated ‘guess’ of the translation vector between test & reference



# HR-EBSD – shifts to strains



$$r_x(A_{xx} - A_{zz}) + r_y A_{xy} + r_z A_{xz} + \frac{r_x r_x}{r_z} A_{zx} + \frac{r_y r_x}{r_z} A_{zy} = Q_x$$

$$r_x A_{yx} + r_y(A_{yy} - A_{zz}) + r_z A_{yz} + \frac{r_x r_y}{r_z} A_{zx} + \frac{r_y r_y}{r_z} A_{zy} = Q_y$$

Infinitesimal:

$$\omega = \frac{1}{2}(A - A^T)$$

$$\varepsilon = \frac{1}{2}(A + A^T)$$

Finite:

$$\varepsilon = \frac{1}{2}(F^T \cdot F - I)$$

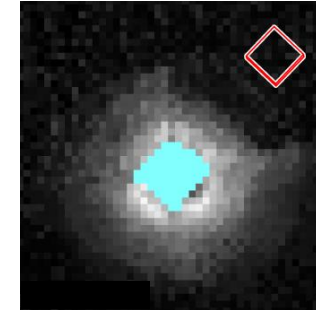
Deformation gradient  $\rightarrow$   $F = A + I = U\Sigma V^*$   $\leftarrow$  Displacement gradient

$\leftarrow$  Finite rotation  $R = UV^*$

# Error Metrics\* - measure precision

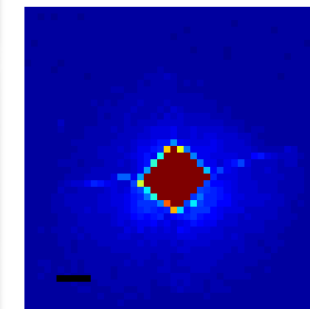
- ◎ Mean Angular Error (MAE)
  - Solve an over determined problem (shifts → deformation)
    - Only 4x regions of interest (ROI) needed
    - Typically use 20+ ROI
  - Tests how well shifts + remapping 'fit' a deformation gradient
  - Need a value < strains of interest
  - Low values can hide systematic errors
  
- ◎ Measure correlation peak height (PH)
  - Normalise with 1 (autocorrelation) and 0 = no correlation
  - Calculate geometric mean → 1 bad ROI reduces PH strongly
  - Typically values >0.3 are 'ok' (higher is better!)

EBSD Local Misorientation Map  
with Crystal Orientation Overlay

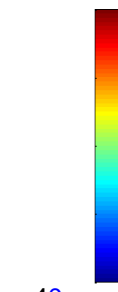


Scale bar = 5μm and step size = 1μm

Mean Angular  
Error (MAE)



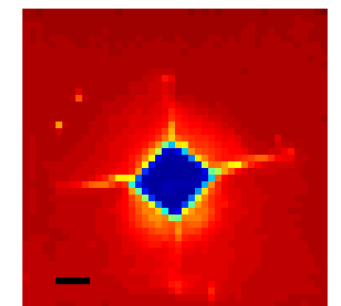
40x10<sup>-4</sup>



40

MAE

Peak Height (PH)



1

0

PH

# Reference selection

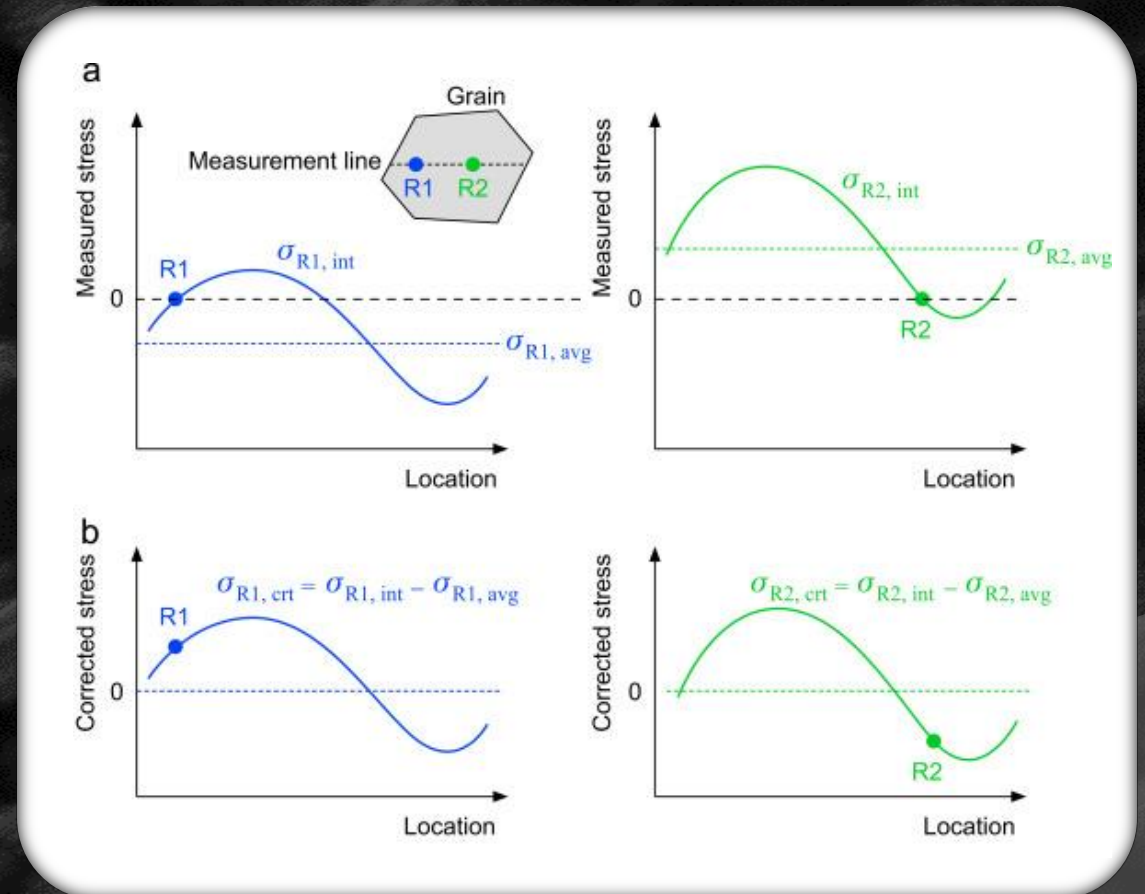
- HR-EBSD measures difference in strain + orientation between test & reference pattern
- Reference within same grain  
& same sample  
& microscope cannot be disturbed
  - $1\mu\text{m}$  of misalignment =  $1/20^{\text{th}}$  pixel =  $1 \times 10^{-4}$  in strain error
- Simulations as reference patterns requires
  - great patterns → dynamical patterns likely ok (e.g. EM Soft + Dynamics)
  - & great knowledge of pattern centre → still not good enough
  - & great knowledge of camera optics → not well tackled (yet?)



# Reference selection

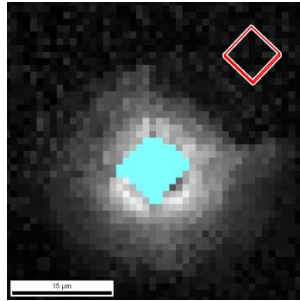
- Choose something clear (single pattern, crisp)
- Precise location does not matter
- Relative strain + rotation measured
- Can 're-zero' as needed
  
- Reference selection for GND measurements not important
  - Local curvature

Illustration of the effect of reference pattern selection  
From Mikami et al. (2015) *Mat Sci Eng A*

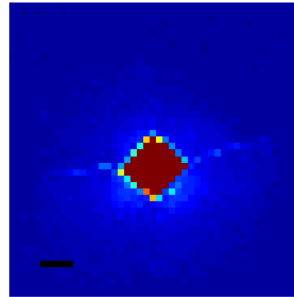


# Example – Si indentation

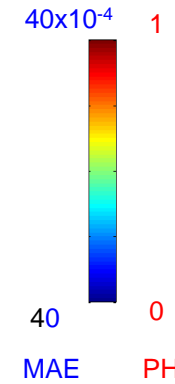
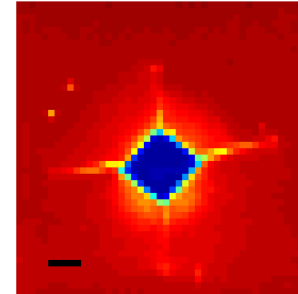
EBSD Local Misorientation Map with Crystal Orientation Overlay



Mean Angular Error (MAE)

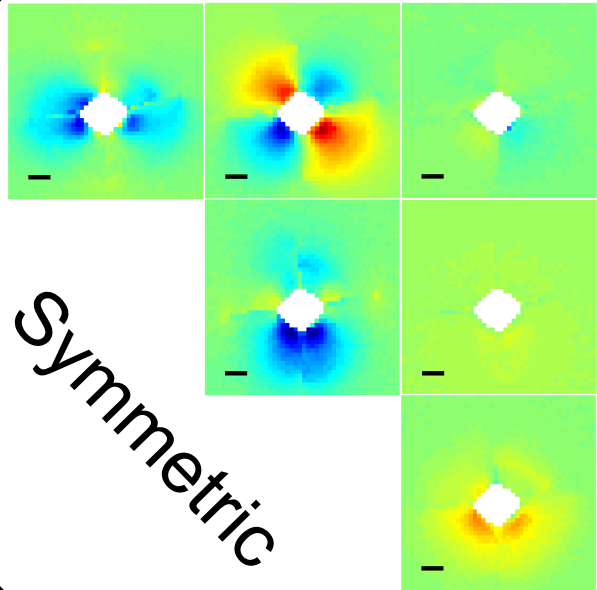


Peak Height (PH)

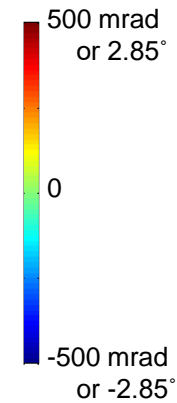
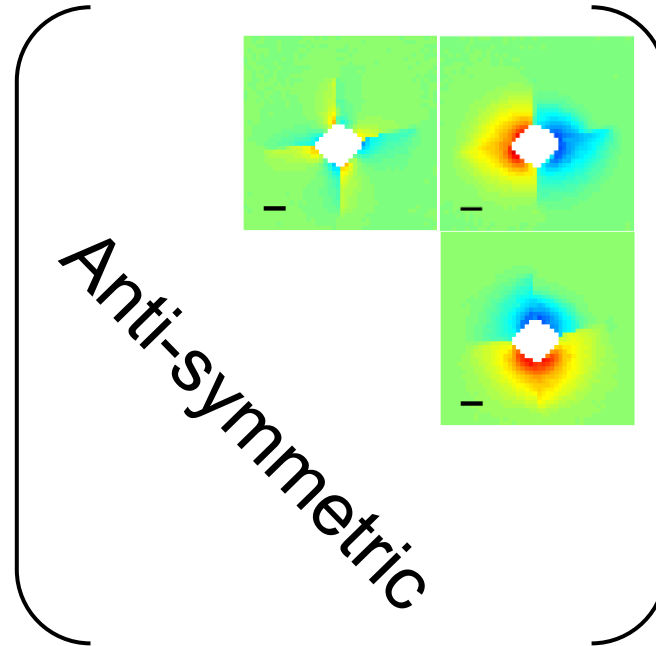
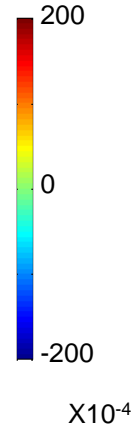


Scale bar = 5 μm and step size = 1 μm

$\epsilon =$



$\omega =$

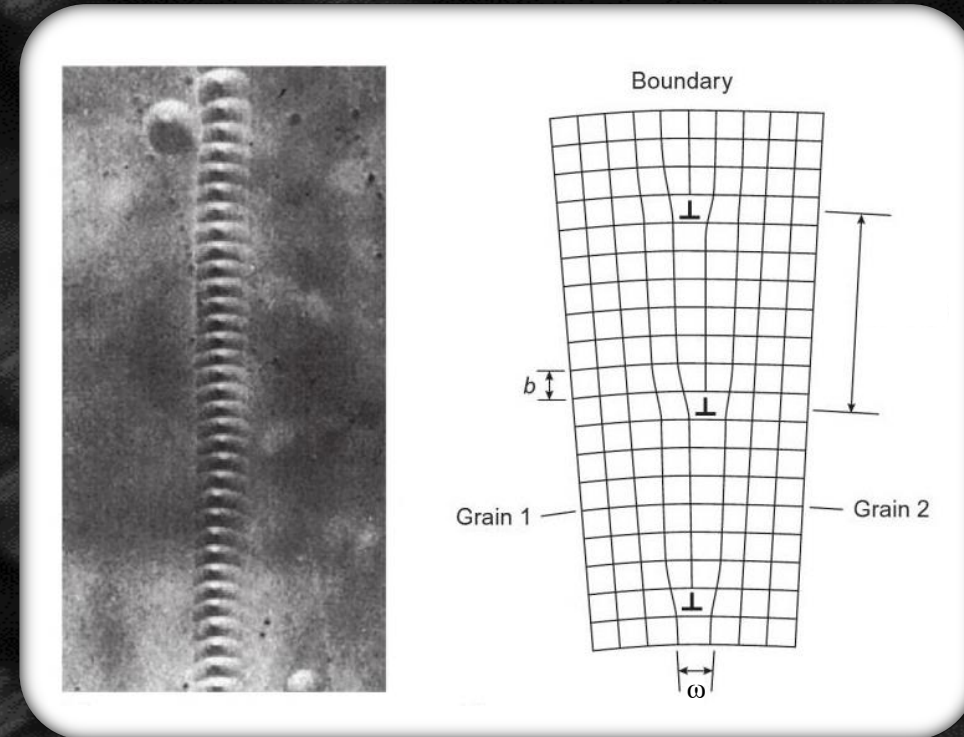


# Closure, Nye Tensor and GNDs

- ⦿ In a deformed crystal (w/o cracks) the displacement field is continuous
- ⦿ Therefore:  $\text{curl}(\mathbf{F}^e) = -\text{curl}(\mathbf{F}^p)$
- ⦿ Evaluate  $\text{curl}(\mathbf{F}^e)$  to understand some plastic strain gradients
- ⦿ Field of GND analysis – using the Nye tensor [1]

# Measuring dislocation content (GNDs)

- Map lattice rotations
- Calculate curvature
  - $K_n = d\omega_{ij}/dx_k$
- Nye's dislocation tensor [1] relates curvature to dislocation content



Etch pits revealing a low angle grain boundary containing an array of geometrically necessary dislocations (GNDs)

[From Hull and Bacon, Introduction to Dislocations]

[1] Nye (1953) *Acta Mat*

# Curvatures & Densities

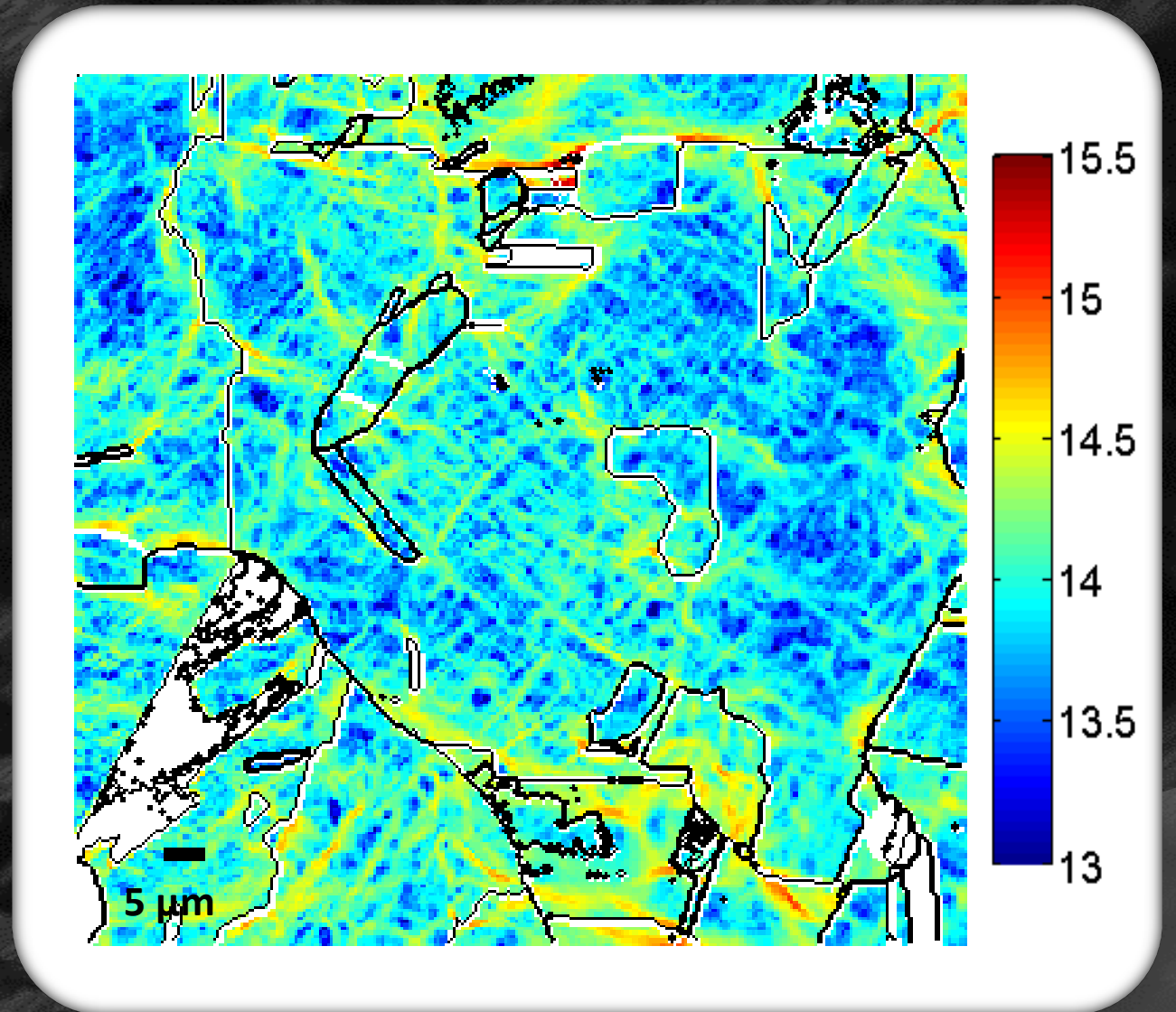
- ⊙ FCC = 18 dislocation types
  - 6 screw, 12 edge
- ⊙ (often\*) overdetermined problem
- ⊙ Solve with physically motivated minimisation:
  - use linprog & weight according to line energies

$$\begin{pmatrix} b_1 l_1 - \frac{1}{2} b \cdot l & \rightarrow \mathcal{S}^{\text{th}} \\ b_1 l_2 \\ b_1 l_3 \\ b_2 l_1 \\ b_2 l_2 - \frac{1}{2} b \cdot l \\ b_2 l_3 \end{pmatrix} \begin{pmatrix} \rho \\ \mathcal{S}^{\text{th}} \end{pmatrix} = \begin{pmatrix} \partial \omega_{23} / \partial x_1 \\ \partial \omega_{31} / \partial x_1 \\ \partial \omega_{12} / \partial x_1 \\ \partial \omega_{23} / \partial x_2 \\ \partial \omega_{12} / \partial x_2 \\ \partial \omega_{23} / \partial x_2 \end{pmatrix}$$

$$A \begin{pmatrix} \rho \\ \mathcal{S}^{\text{th}} \end{pmatrix} = K$$

# Example map

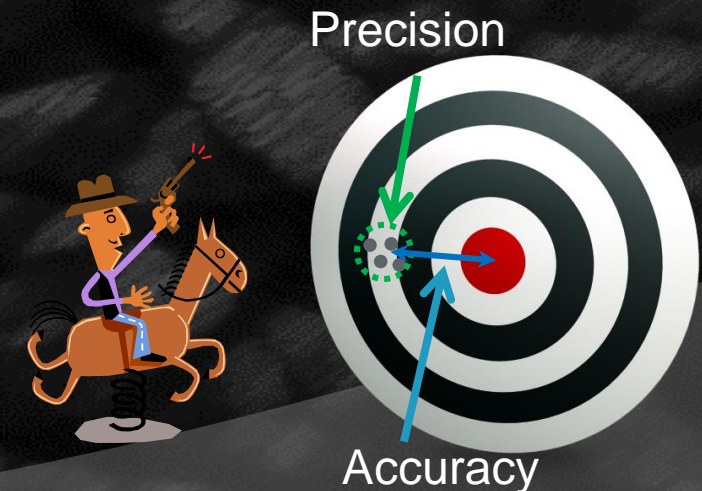
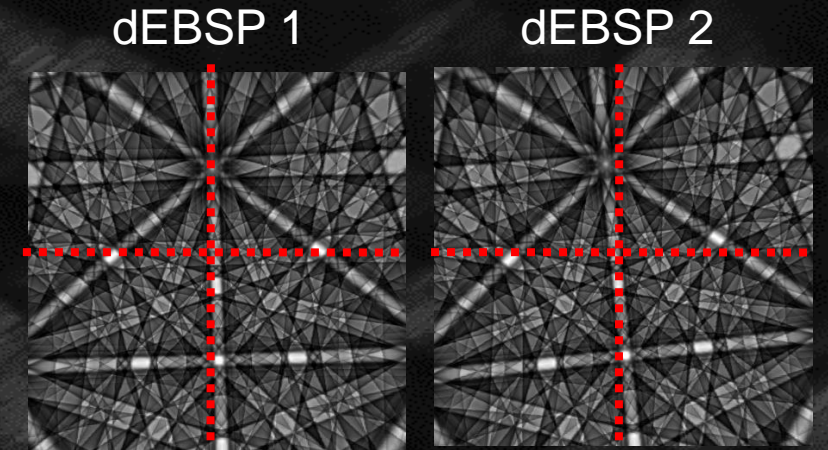
- Cu – 2% plastic strain
- Reveals cell structure
- Relationship of GND distributions with microstructure...



# Accuracy, Precision & Sensitivity

# Accuracy, Precision and Sensitivity

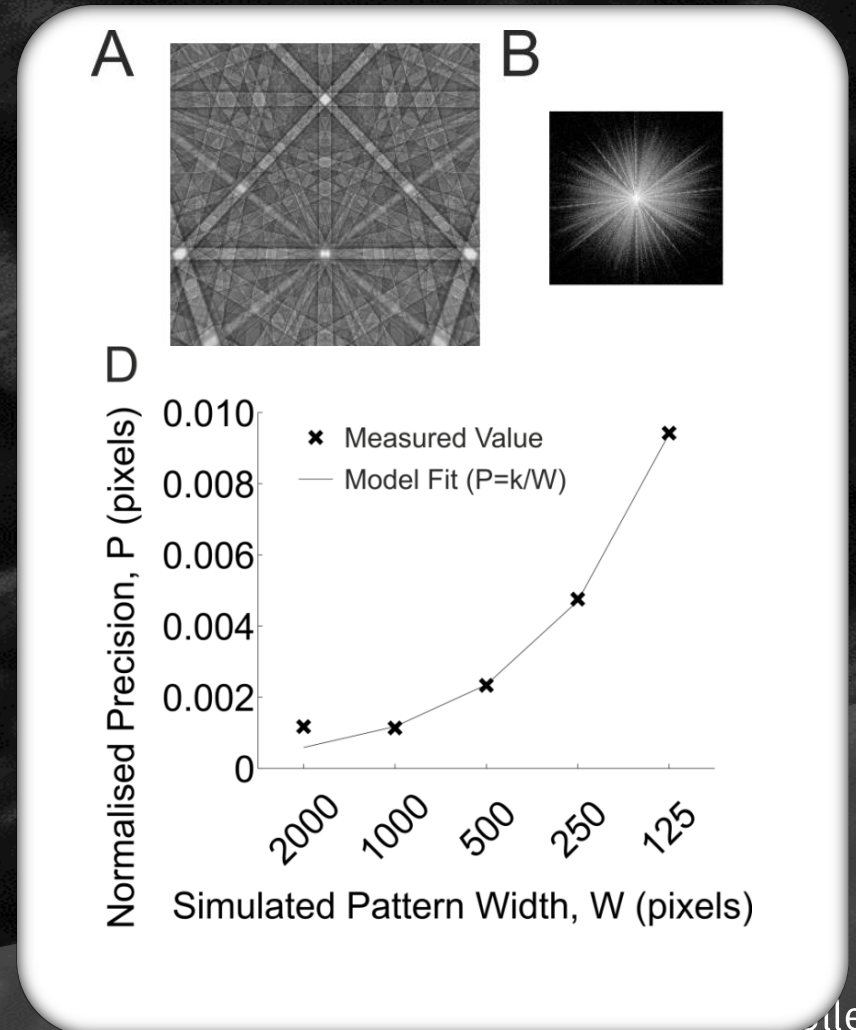
- Fundamentally image correlation used to compare diffraction patterns and extract 'high quality' data
  - Precision
    - ability to recover same result many times
  - Accuracy
    - ability to recover correct answer
  - Sensitivity
    - what sort of changes can we observe





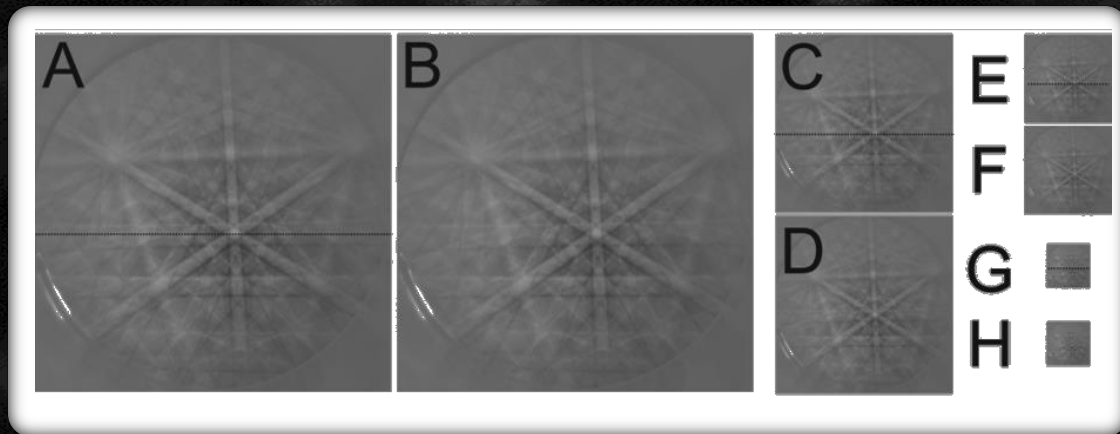
# Measuring Precision – Dynamical Simulations

- Generate a high quality 4MG simulation
  - ecpdist courtesy of Dr Aimo Winklemann
- Beam shift virtually
- Bin the image
- Measure (normalised) precision for each binning level

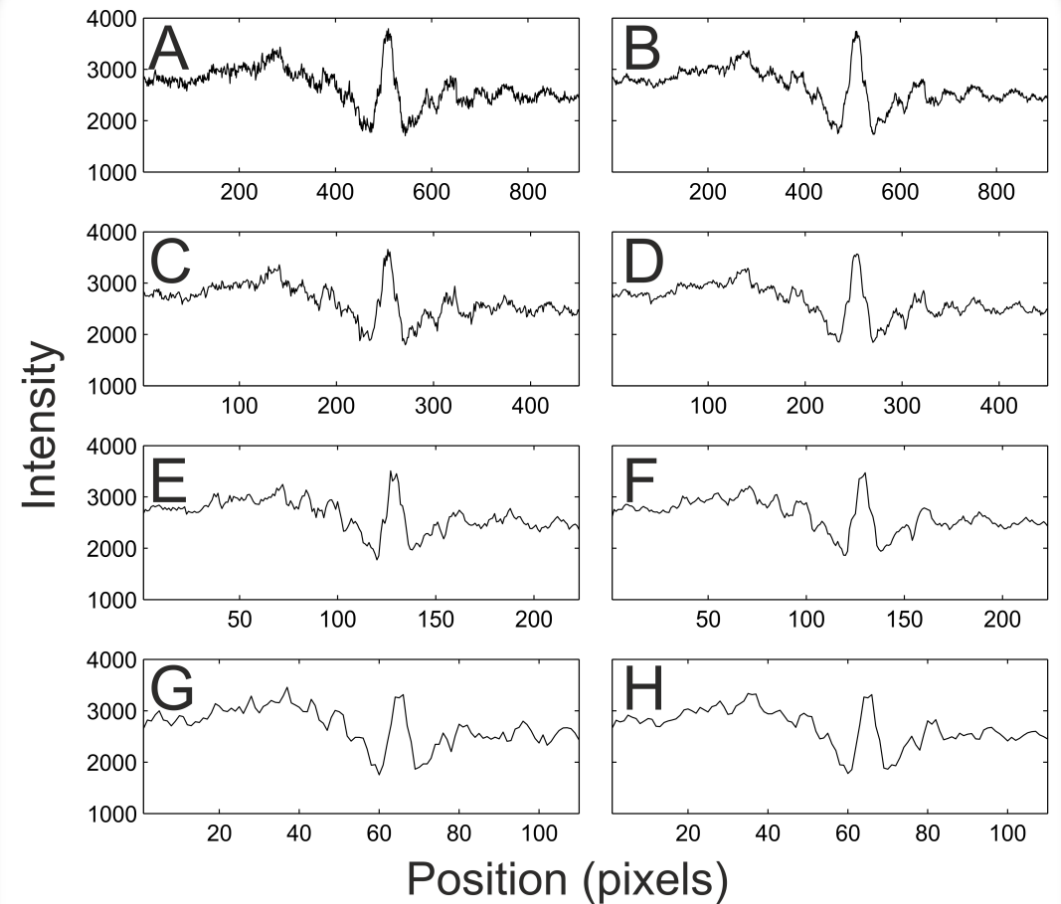


# Capturing Diffraction Patterns

- ◎ Variables:
  - Hardware (~fixed)
  - Exposure time/probe current
    - i.e. electron budget
  - Camera binning

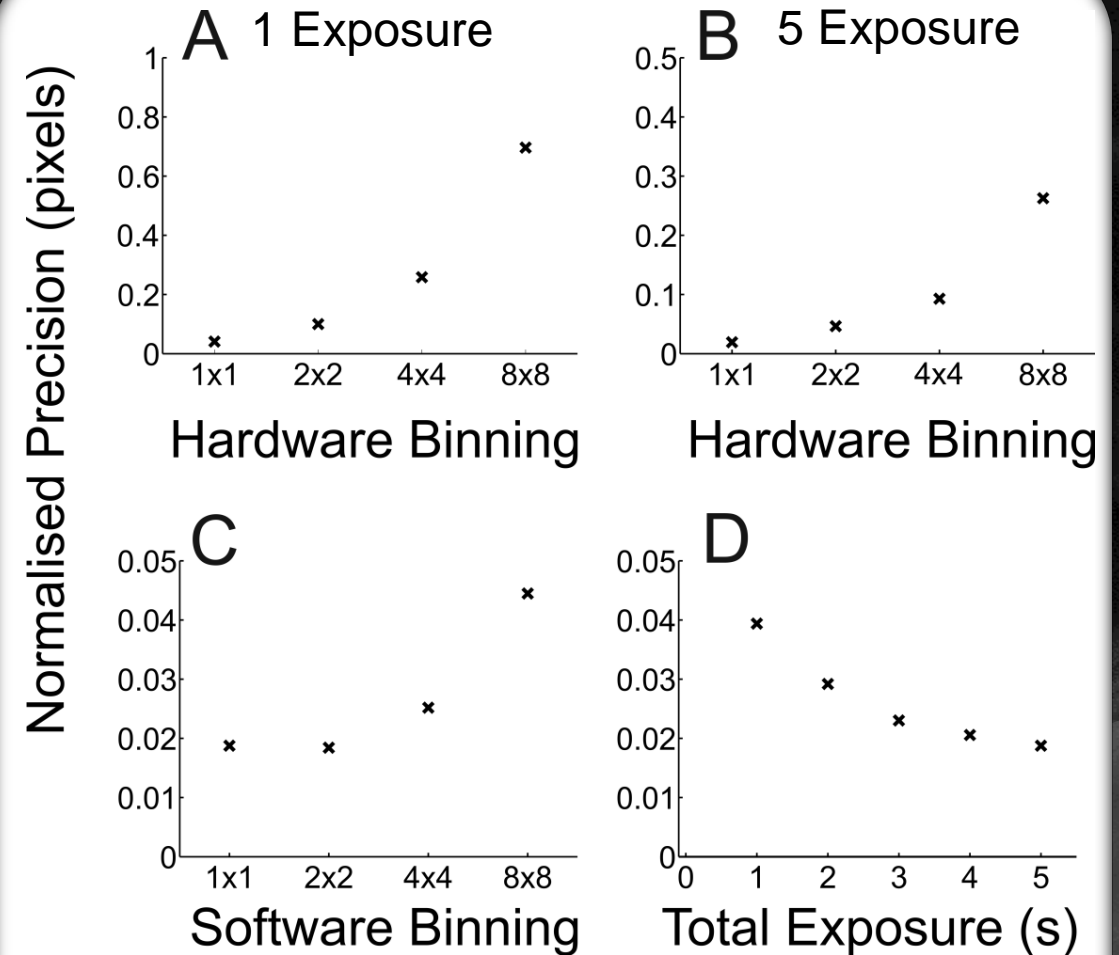


One Pattern → Five Patterns



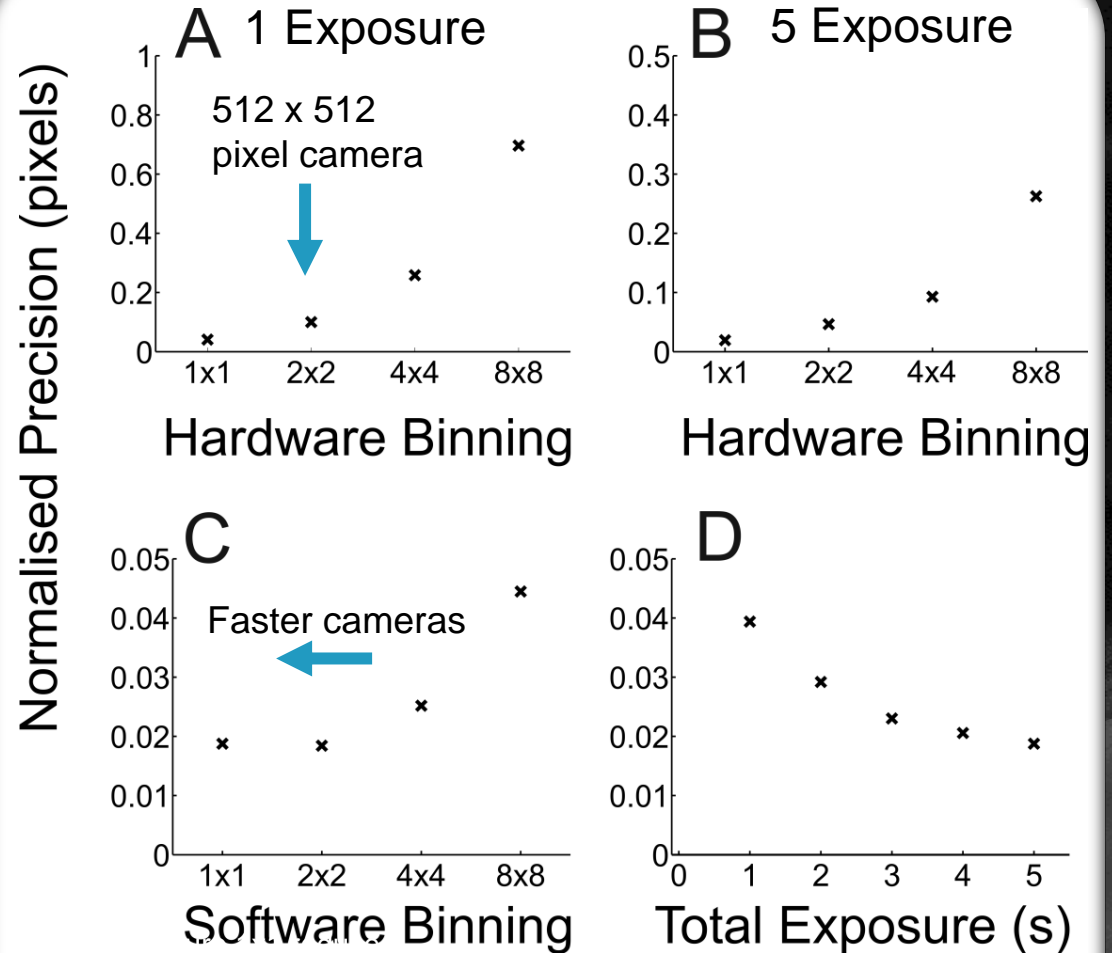
# Precision vs budget and binning

- Measure variation EBSP movement due to beam shift
  - 10 pairs of patterns
  - Different binning (hardware & software)
  - Different exposure times
- Measure precision
  - Averaging standard deviation of 50 ROIs for the difference



# Implications

- ⦿ Sensitivity proportional to precision
- ⦿ 0.1 normalised pixels in 1000 pixels  
~  $1 \times 10^{-4}$  in sensitivity\*
- ⦿ 2x2 binning, faster exposures or 'fast cameras' ok!
  - Hough resolution:
    - $0.5^\circ = 8$  pixels shift
  - Just 0.5 pixels shift sensitivity?
    - $0.02^\circ$  misorientation resolution!



# HR-EBSD resolution & precision

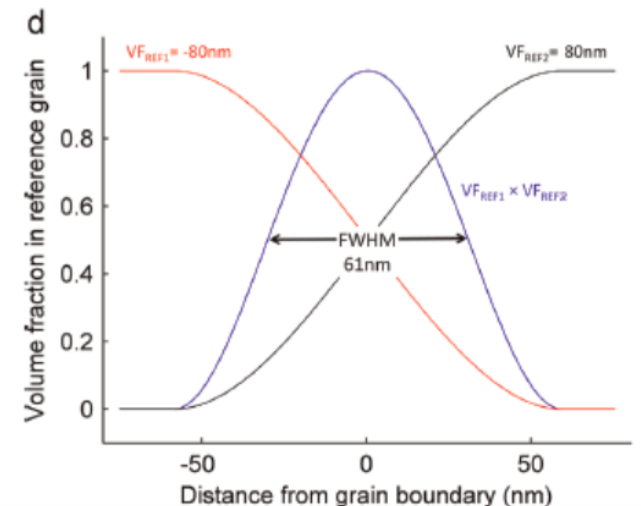
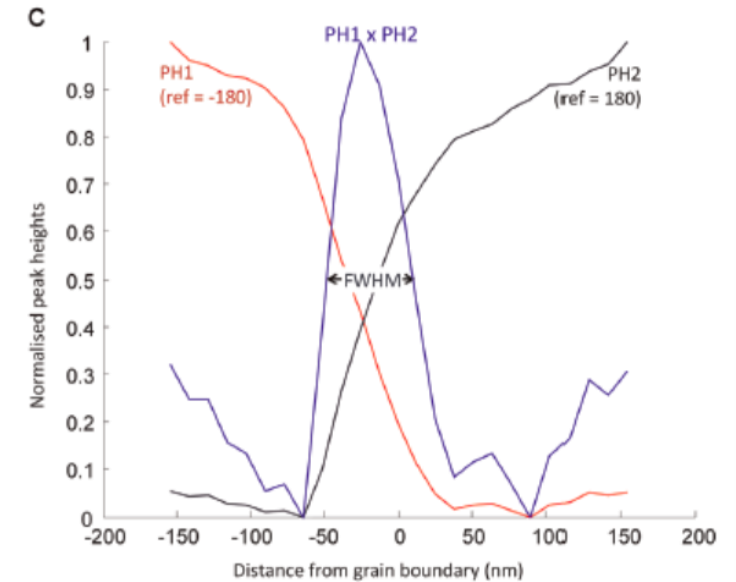
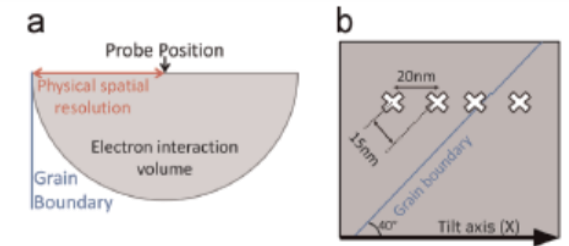
- (Effective) spatial resolution – ~typical EBSD
  - c.  $20 \times 60 \times 20 \text{ nm}^3 \rightarrow$  see figure to right [1]
- Typical precision [2,3]
  - $\sim 1 \times 10^{-4}$  in strain
  - $\sim 1 \times 10^{-4} / 0.006^\circ$  in rotation
    - vs Hough at  $0.8 \times 10^{-2} / 0.5^\circ$
    - Up to  $\sim 1 \times 10^{-5}$  reported with great patterns
- Could be better with better hardware... [4]

[1] Tong, Jiang, Wilkinson and Britton (2015) Ultramicroscopy

[1] Wilkinson, Meaden and Dingley (2006) Ultramicroscopy

[2] Britton, Jiang, Karamched, Wilkinson (2013) JOM

[3] Britton et al. (2013) "Assessing the precision of strain measurements using electron backscatter diffraction" – Parts 1 and 2" in *Ultramicroscopy*



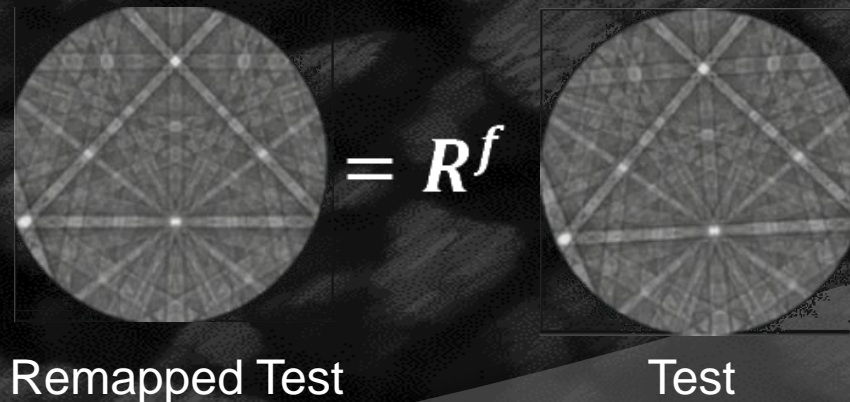
# HR-EBSD: Remapping

# Pattern remapping

- 3° rotation → ~10s of pixels shifts
- 0.001 strain → ~1s pixel shifts
- Estimate finite rotation matrix,  $R^f$ , from measured (infinitesimal) rotations
- Interpolate test pattern into reference orientation
  - Bicubic in matlab (gridfit)

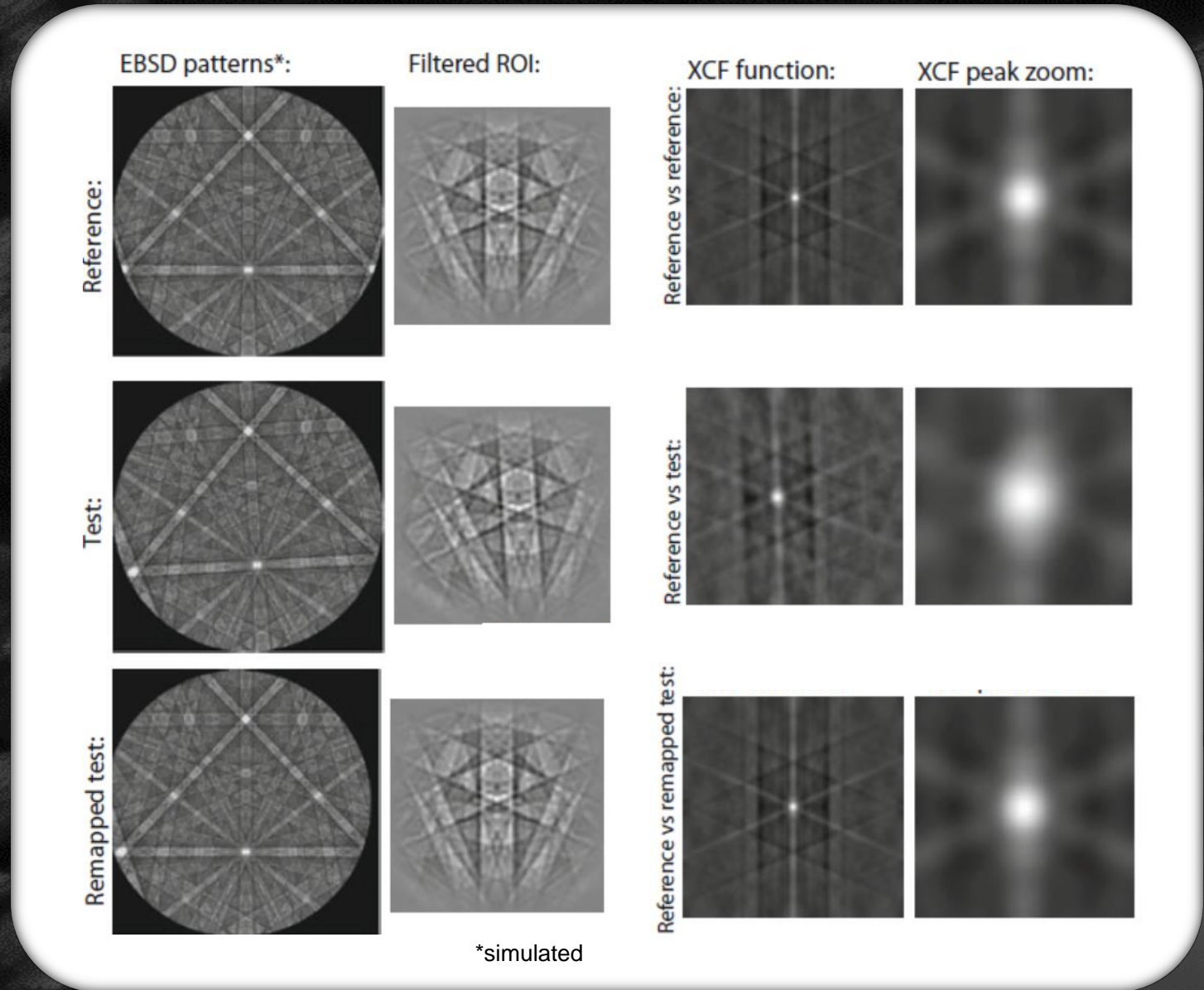
$$R^f = \begin{pmatrix} \cos \omega_{12} & \sin \omega_{12} & 0 \\ -\sin \omega_{12} & \cos \omega_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \omega_{23} & \sin \omega_{23} \\ 0 & -\sin \omega_{23} & \cos \omega_{23} \end{pmatrix} \times \begin{pmatrix} \cos \omega_{31} & 0 & -\sin \omega_{31} \\ 0 & 1 & 0 \\ \sin \omega_{31} & 0 & \cos \omega_{31} \end{pmatrix}$$

$$\mathbf{r}' = R^f \mathbf{r}$$



# Remapping

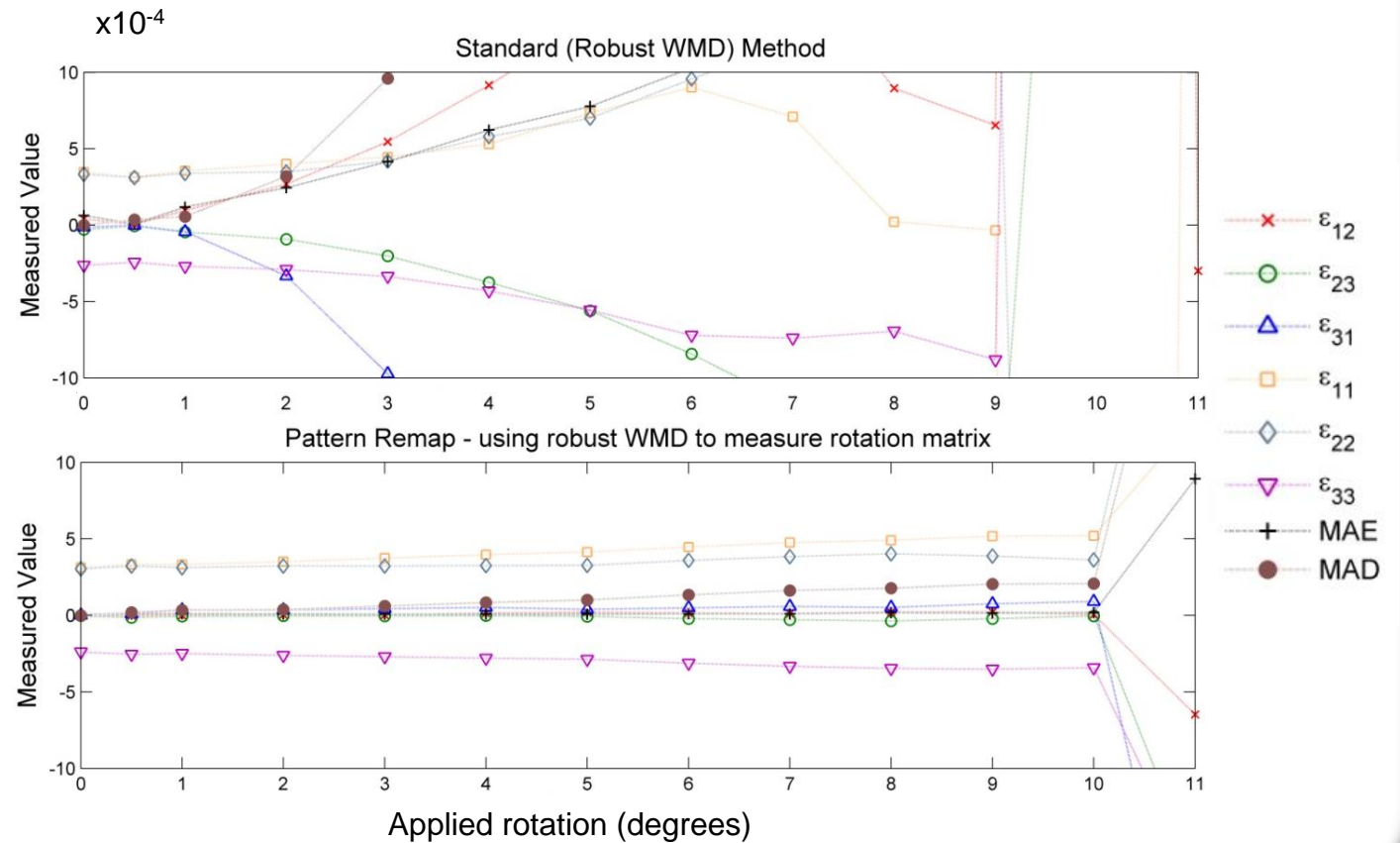
- 1<sup>st</sup> pass measurement of rotation
- Use to remap intensities
  - Need projection info
  - Improve XCF
- Cross correlation for strains + precise rotations
- Recombine the maths in finite deformation framework



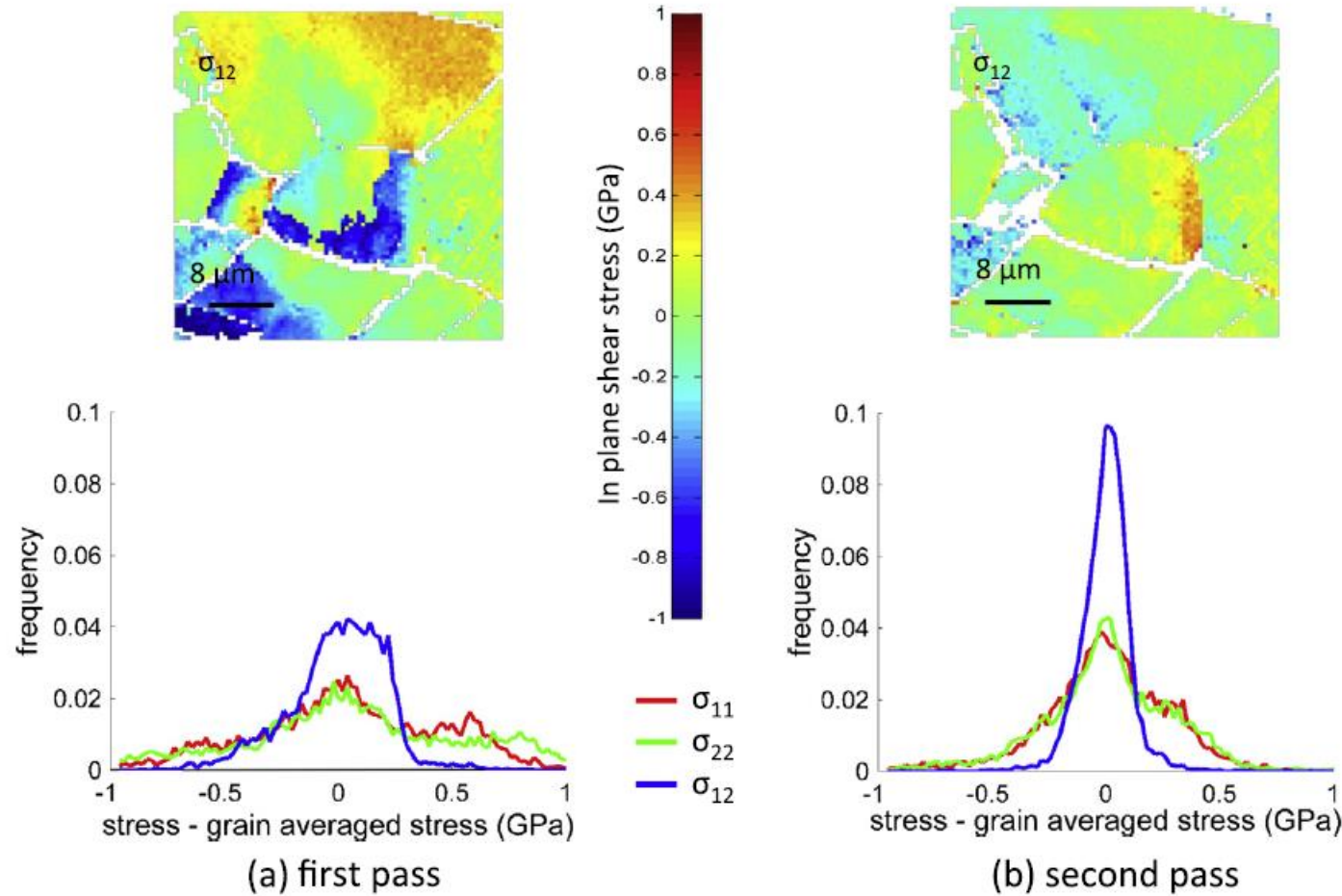


# Measuring strains with large rotations

- Comparing simulated patterns (tetragonal distortion)
- Errors introduced when rotations large
- Due to non-translational distortions
- Use pattern remapping to fix
- Needs a good pattern centre measurement!



# More reasonable stresses?



# HR-EBSD: Summary

# Summary

- Compare 2+ good patterns ... many many times (an embarrassing problem)
- Software evaluates  $F^e$
- Explore residual elastic strain tensor (i.e. stress) within grains
- Measure lattice rotation gradients → enables measurement of GND content via Nye's analysis
  - Only a symptom of part of plastic strain...
- Use of simulations, at present, is limited to algorithm development\*